Long Term Evolution of Close Planets Including the Effects of Secular Interactions

Fred C. Adams[1,2] and Gregory Laughlin[3]

[1]*Michigan Center for Theoretical Physics Physics Department, University of Michigan, Ann Arbor, MI 48109*

[2]*Astronomy Department, University of Michigan, Ann Arbor, MI 48109*

[3]*Lick Observatory, University of California, Santa Cruz, CA 95064*

ABSTRACT

This paper studies the long term evolution of planetary systems containing short-period planets, including the effects of tidal circularization, secular excitation of eccentricity by companion planets, and stellar damping. For planetary systems subject to all of these effects, analytic solutions (or approximations) are presented for the time evolution of the semi-major axes and eccentricities. Secular interactions enhance the inward migration and accretion of hot Jupiters, while general relativity tends to act in opposition by reducing the effectiveness of the secular perturbations. The analytic solutions presented herein allow us to understand these effects over a wide range of parameter space and to isolate the effects of general relativity in these planetary systems.

Subject headings: Stars: Planetary systems

1. Introduction

Starting with the discovery of extrasolar planets (Mayor & Queloz 1995; Marcy & Butler 1996), a substantial fraction of the planetary orbits have been found close to their stars, with periods $P \sim 4$ days. These objects are often referred to as "hot Jupiters". With ~ 200 planets detected to date[1], the distribution of orbital periods shows a measurable pile-up at periods $P = 3$–5 days, i.e., roughly 10 percent of the currently detected planets have periods $P < 5$ days. Although the close planets are the most easily detected, this finding is not a selection effect: The current statistics indicate that 1.2 percent of all FGK stars have hot Jupiters within 0.1 AU of their stars (Marcy et al. 2005). These hot Jupiters are subject to tidal interactions with their central stars (e.g., Goldreich & Soter 1963), and this star-planet coupling can influence the long term evolution of these systems. If the system contains additional bodies, then planet-planet interactions can also affect the long term

[1]http://www.ucolick.org/~ laugh/

evolution of the close planets. In our companion paper (Adams & Laughlin 2006a; hereafter Paper I), we present a treatment of secular interactions for (non-resonant) systems with multiple planets. In this paper, we combine this formulation of secular interactions with star-planet interactions to study the long term fate and evolution of close planets.

The basic theory of secular interactions, as applied to extrasolar planetary systems and used herein, is reviewed in Paper I (see also Murray & Dermott 1999; hereafter MD99). These interactions allow for planetary orbits to exchange angular momentum so that orbital eccentricities change on secular time scales that are long compared to both the orbit time and observational monitoring times (tens of years), but short compared to the ages of the systems (a few Gyr). The characteristic secular interaction time scales provide a simple metric of the importance of planet-planet perturbations within a given system. For a collection of observed multi-planet systems, these secular time scales fall in the range 100 – 50,000 yr, as listed in Table 1 of Paper I. On longer time scales, tidal interactions with the star act to circularize close orbits and can lead to continued inward migration. This process might have a bearing on the observed pile-up of planets with periods of 3–4 days and can lead to considerable energy input into the planetary atmospheres. Although this process has been considered previously (e.g., Trilling 2000; Bodenheimer et al. 2001, 2003; Yu & Goldreich 2002; Mardling & Lin 2002, 2004; Gu et al. 2003; Faber et al. 2005), this paper takes the additional step of providing analytic expressions for the evolution time for various classes of systems, including both tidal damping and secular interactions. This work thus provides additional analytic insight into the problem as well as elucidating the role played by secular interactions. The version of secular interaction theory formulated in Paper I includes the leading order corrections for general relativity (GR), which causes the periasta of planets to precess forward in their orbits. As a result, this semi-analytical treatment also allows us to explicitly delineate the role played by general relativity in the long term evolution of close planets.

For a single planet in a close orbit, the tidal circularization effect is usually written in terms of the time scale for eccentricity decay, $\tau_{cir} = -e/\dot{e}$. For close orbits in multiple planet systems, however, the eccentricity is excited via interactions that we model according to secular theory. The first approximation to the long term behavior is to assume that the orbit decays with constant angular momentum and that the semi-major axis decreases on the same time scale, i.e.,

$$\frac{\dot{a}}{a} = \frac{2e\dot{e}}{1-e^2} = -\frac{2e^2}{(1-e^2)}\frac{1}{\tau_{cir}}\,, \tag{1}$$

where the time scale τ_{cir} for circularization can be written in the form

$$\tau_{cir} \approx \frac{4Q_P}{63}\Big(\frac{a^3}{GM_*}\Big)^{1/2}\frac{m_P}{M_*}\Big(\frac{a}{R_P}\Big)^5(1-e^2)^{13/2}[F(e^2)]^{-1}\,, \tag{2}$$

where $Q_P \approx 10^5 - 10^6$ is the tidal quality factor and R_P is the planet radius. For extrasolar planets, the quality factor Q_P and the radius R_P depend sensitively on the planetary mass, temperature, and composition (Bodenheimer et al. 2003). This general form for the time circularization scale is well known (e.g., Goldreich & Soter 1966), but includes additional factors to account for the effects

of nonzero eccentricity (see Hut 1981). The choice $F \approx 1 + 6e^2 + \mathcal{O}(e^4)$ provides a reasonable approximation to the results for close binaries (Hut 1981) for moderate eccentricities. Inserting representative values, and taking the limit $e \to 0$, we write the circularization time scale in the form

$$\tau_0 = \tau_{cir}(e=0) \approx \ 1.6\,\mathrm{Gyr} \left(\frac{Q_P}{10^6}\right) \left(\frac{m_P}{m_J}\right) \left(\frac{M_*}{M_\odot}\right)^{-3/2} \left(\frac{R_P}{R_J}\right)^{-5} \left(\frac{a}{0.05\,\mathrm{AU}}\right)^{13/2} . \tag{3}$$

Throughout this paper we work in terms of the dimensionless time scale

$$\hat{t} \equiv t/\tau_0 \,. \tag{4}$$

These systems are also affected by a stellar damping effect in which energy is dissipated in the star due to tides raised by the planet. The effectiveness of this process is determined by an analogous parameter Q_*, the tidal quality factor of the star. The net result of this process is to cause the semi-major axis of the inner planet to decay on a time scale τ_{P*} (again, see Goldreich & Soter 1966, Hut 1981) that can be written in the form

$$\tau_{P*} = \tau_0 \Gamma^{-1} \left(\frac{a}{a_0}\right)^{13/2} , \qquad \text{where} \qquad \Gamma \equiv \frac{2}{7} \left(\frac{R_*}{R_P}\right)^5 \left(\frac{Q_P}{Q_*}\right) \left(\frac{m_P}{M_*}\right)^2 . \tag{5}$$

As defined here, Γ is generally a small parameter. For Jupiter-like planets, $R_P \sim 0.1 R_*$ and $m_P \sim 10^{-3} M_*$ so that $\Gamma_J \sim 0.03(Q_P/Q_*)$. For Neptune-like planets, we find $\Gamma_{Nep} \sim 0.015(Q_P/Q_*)$, but we expect the Q_P value to be much smaller so that $\Gamma_{Nep} \ll \Gamma_J$.

This paper considers hot Jupiters subject to tidal circularization effects (with strength determined by Q_P), both in single planet systems (§2.1) and in two-planet systems where the eccentricity of the inner planet is excited through secular interactions (§2.2). Next we consider hot Jupiter systems with additional stellar damping effects (with strength determined by Q_*), both for single planet systems (§3.1) and two-planet systems (§3.2). Our results are summarized in §4 along with a discussion of their ramifications.

2. Long Term Evolution with No Stellar Damping

The basic goal of this section is to provide an analytic understanding of the long term behavior of the hot Jupiter systems in the absence of stellar damping terms (note that stellar damping is included in the following section). For single planet systems that experience tidal forces only, we define $g(t) \equiv a_1(t)/a_1(0)$. Similarly, for two planet systems, we define $f(t) \equiv a_1(t)/a_1(0)$, where the inner planet experiences circularization from the central star and eccentricity excitation from the other planet.

2.1. One planet system with no stellar damping

For a one planet system, the evolution is described by two coupled equations of motion. If we work to the same order of approximation in e^2, these equations take the form

$$\frac{dg}{d\hat{t}} = -2e^2 g^{-11/2} \qquad \text{and} \qquad \frac{de}{d\hat{t}} = -e g^{-13/2}\,, \tag{6}$$

which be be combined to form the second order differential equation

$$(\frac{dg}{d\hat{t}})^{-1}\frac{d^2g}{d\hat{t}^2} + \frac{11}{2g}\frac{dg}{d\hat{t}} = -2g^{-13/2}\,. \tag{7}$$

The first integral of this equation can also be found, i.e.,

$$\frac{dg}{d\hat{t}} = -2g^{-11/2}(e_{1(0)}^2 + \ln g)\,, \tag{8}$$

where $e_{1(0)}$ is the eccentricity of the inner planet at $t = 0$. In contrast to the two planet case, where continued eccentricity forcing by the second planet leads to continued evolution, this system asymptotically approaches a minimum value of semi-major axis, i.e., a minimum value of g given by $g_\infty = \exp[-e_{1(0)}^2]$. Combining the two expressions in equation (6), we can also solve directly for the eccentricity as a function of the factor g, i.e., $e_1^2 = e_{1(0)}^2 + \ln g$. This expression does not satisfy conservation of angular momentum exactly because of the approximation made at the start (where we work to only leading order in e^2).

If we include the additional factors to enforce conservation of angular momentum, the equations of motion take the form

$$\frac{dg}{d\hat{t}} = -2\frac{e^2}{1-e^2}g^{-11/2} \qquad \text{and} \qquad \frac{de}{d\hat{t}} = -e g^{-13/2}\,, \tag{9}$$

and the resulting second order differential equation becomes

$$(\frac{dg}{d\hat{t}})^{-1}\frac{d^2g}{d\hat{t}^2} + \frac{11}{2g}\frac{dg}{d\hat{t}} = -2g^{-13/2}\frac{g}{1-e_0^2}\,. \tag{10}$$

Here we have eliminated the eccentricity dependence using conservation of angular momentum, i.e., $g(1-e^2) = (1-e_0^2) = J^2 = \mathit{constant}$. This second order differential equation can be integrated once to find the time as a function of g, i.e.,

$$\hat{t} = \frac{J^2}{2}\int_g^1 \frac{g^{11/2}dg}{g-J^2}\,. \tag{11}$$

The integral can also be evaluated to find the implicit solution

$$\begin{aligned}\hat{t} = {} & \frac{J^2}{11}(1-g^{11/2}) + \frac{J^4}{9}(1-g^{9/2}) + \frac{J^6}{7}(1-g^{7/2}) + \frac{J^8}{5}(1-g^{5/2}) + \\ & \frac{J^{10}}{3}(1-g^{3/2}) + J^{12}(1-g^{1/2}) + \frac{1}{2}J^{13}\ln\left|\frac{1-J}{1+J}\cdot\frac{g^{1/2}+J}{g^{1/2}-J}\right|\,.\end{aligned} \tag{12}$$

Both the time integral (eq. [11]) and the solution (eq. [12]) indicate that the planet only reaches its final location (given by $g = J^2$) asymptotically in time ($t \to \infty$).

2.2. Two planet system with no stellar damping

For the long term evolution of a two planet system, the eccentricity of the inner planet is driven by the interactions with the outer planet. Within this set of approximations, the evolution is given by the time-averaged equation

$$\frac{\dot{f}}{f} = -2\langle e^2 \rangle \, \langle \frac{1}{\tau_{cir}} \rangle = -2\langle e^2 \rangle \, f^{-13/2} \,, \tag{13}$$

where the angular brackets denote time averages over an intermediate time scale that is long compared to the secular time scales (e.g., see Table 1 of Paper I) and short compared to the circularization time scales (eq. [3]). As the system evolves, the semi-major axis of the inner planet decreases, which in turn causes the secular averaged square eccentricity $\langle e^2 \rangle$ to decrease. Notice that by time-averaging the square of the eccentricity and the circularization time scale as separate quantities, we are making an approximation that limits the accuracy of this treatment to $\mathcal{O}(e^2)$. In practice, however, we evaluate τ_{cir} in the $e \to 0$ limit, and the secular interaction theory used here is only accurate to second order in e (Paper I; MD99), so that this order of approximation is consistent with our general framework.

In the absence of other effects, the equation of motion for the two planet system can be written in the form

$$\frac{df}{d\hat{t}} = -4\eta^2 f^{-11/2} \,, \tag{14}$$

where the eccentricity excitation amplitude η takes the approximate form

$$\eta^2 \approx \frac{25}{16} \frac{e_{2(0)}^2 \alpha_0^2 f^2}{(1 + \Pi_0 f^{-3} - \delta\sqrt{\alpha_0 f})^2 + (25/4)\delta\alpha_0^{5/2} f^{5/2}} \,, \tag{15}$$

where we have defined $\delta \equiv m_1/m_2$ and $\alpha_0 \equiv a_1(0)/a_2(0)$. The dimensionless parameter $\Pi_0 = 4GM_*^2 a_2^3/(m_2 c^2 a_1^4)$ provides a measure of the importance of general relativity in secular interactions, where the semi-major axes are evaluated at $t = 0$ (from eq. [17] of Paper I; see also Adams & Laughlin 2006b). The quantity $e_{2(0)}$ is the eccentricity of the second planet evaluated at $t = 0$ (although this eccentricity is assumed constant in this set of approximations). It is useful to collect the constants into the dimensionless composites

$$A \equiv \frac{25}{4} e_{2(0)}^2 \alpha_0^2 \,, \qquad B \equiv \delta\sqrt{\alpha_0} \,, \qquad \text{and} \qquad C \equiv \frac{25}{4}\delta\alpha_0^{5/2} \,. \tag{16}$$

With these definitions, the equation of motion can be written in the form

$$\frac{df}{d\hat{t}} = -\frac{A f^{5/2}}{(\Pi_0 + f^3 - B f^{7/2})^2 + C f^{17/2}} \,. \tag{17}$$

By absorbing the leading coefficient A into the time variable, we can define the relevant time scale τ_S for the evolution of this system, i.e.,

$$\tau_S \equiv \frac{4\tau_0}{25\alpha_0^2 e_{2(0)}^2} = \frac{\tau_0}{A} \,. \tag{18}$$

This time scale is often much longer than the circularization time τ_0 that applies for single planet systems. For example, using the observed parameters for the inner two planets of the Ups And system, we find $A \approx 0.0022$ so that $\tau_S/\tau_0 \approx 454$.

The equation of motion (17) can be integrated to provide the solution in implicit form

$$A\hat{t} = \frac{2}{9}(1 - f^{9/2}) + \frac{4}{3}\Pi_0(1 - f^{3/2}) + \frac{2}{3}\Pi_0^2(f^{-3/2} - 1) + \frac{C}{7}(1 - f^7)$$

$$+\frac{2}{11}B^2(1 - f^{11/2}) - \Pi_0 B(1 - f^2) - \frac{2}{5}B(1 - f^5)\,. \tag{19}$$

In the limit $B, C \ll 1$ (which often holds), this solution can be simplified further to the form:

$$A\hat{t} = \frac{2}{9}(1 - f^{9/2}) + \frac{4}{3}\Pi_0(1 - f^{3/2}) + \frac{2}{3}\Pi_0^2(f^{-3/2} - 1)\,. \tag{20}$$

From this expression, we can read off the asymptotic behavior. As the time $\hat{t}$ increases, $f \ll 1$, and the third term in the equation dominates. As a result, as $\hat{t} \to \infty$, the function $f(\hat{t})$ approaches the limiting form

$$f(\hat{t}) \;\to\; \Pi_0^{4/3}\,(3A\hat{t}/2)^{-2/3}\,. \tag{21}$$

Unlike the case of a single planet system (see §2.1), the migrating planet can continue to lose energy and can become arbitrarily close to the origin. In practice, however, once the planet reaches the stellar surface where $f = f_{min} = R_*/a_1(0)$, the planet will be destroyed and evolution will be over. The time required for the planet to reach the stellar surface is thus given by

$$A\hat{t}_* = \frac{2}{9}(1 - f_{min}^{9/2}) + \frac{4}{3}\Pi_0(1 - f_{min}^{3/2}) + \frac{2}{3}\Pi_0^2(f_{min}^{-3/2} - 1)\,. \tag{22}$$

Notice the important role played by general relativity in this setting. In the absence of relativistic corrections, the (leading order) solution would have the form $f(\hat{t}) = (1 - 9A\hat{t}/2)^{2/9}$, which reaches $f = 0$ in the relatively short time $\hat{t}_* = 2/9A$. Relativistic precession thus acts to keep the planet from being accreted by the star. This claim can be quantified by inserting typical values of interest; let $a_1(0) = 0.05$ AU and $R_* = 1.0\ R_\odot$ so that the total evolution time given by equation (22) can be written $9A\hat{t}_*/2 \approx 1 + 6\Pi_0 + 105\Pi_0^2$. With these values, e.g., the condition for relativistic effects to dominate becomes $\Pi_0 \gtrsim 0.073$.

The portion of parameter space for which two planet systems lead to significant (short) accretion times is depicted in Figure 1. In this application, we assume that the inner planet is a hot Jupiter, with mass $m_1 = m_J$, radius $R_P = R_J$, and starting semi-major axis $a_1 = 0.05$ AU. The eccentricity of the inner planet cycles through $e_1 = 0$, although $\langle e \rangle \neq 0$ due to secular interactions with the second planet. The stellar mass $M_* = 1.0\ M_\odot$ and the tidal quality factor $Q_P = 10^6$ so that the circularization time scale $\tau_0 = 1.6$ Gyr (see eq. [3]). The figure shows the orbital elements of the second planet (a_2, e_2) required to drive the hot Jupiter into the stellar photosphere over a fiducial time scale of 5 Gyr. The accretion time depends on the mass of the second planet, which is assumed to vary over the range $m_2 = 1 - 5m_J$, corresponding to the five solid curves shown in

Figure 1. The region of the $a-e$ plane above the curves represents solar systems in which the inner planet would be driven into the star on time scales less than 5 Gyr. The dashed curve delineates the (much larger) region of parameter space for which the inner planet would be driven into the star in the absence of general relativity (where $m_2 = m_J$). Comparison of the dashed curve with the upper solid curve thus illustrates how GR acts to prevent the accretion of planets.

3. Long Term Evolution with Stellar Damping Term

Given the solutions discussed above, the next correction is to consider the dissipation of energy that occurs due to the planet raising tides on the star. This section presents solutions for the long term evolution of these systems, including both tidal circularization (due to energy dissipated in the planet via Q_P) and orbital damping (due to energy dissipated in the star via Q_*). We present solutions for single planets systems (§3.1) and for two planet systems in which the bodies are also subject to secular interactions (§3.2).

3.1. One planet system with stellar damping term

In the presence of stellar damping, the equations of motion for the one planet system take the form

$$\frac{dg}{d\hat{t}} = -2\frac{e^2}{1-e^2} g^{-11/2} - \Gamma g^{-11/2} \qquad \text{and} \qquad \frac{de}{d\hat{t}} = -e g^{-13/2} \,. \tag{23}$$

In the limit of zero eccentricity, the stellar damping term acts to decrease the semi-major axis of the planet according to $g(\hat{t}) = (1 - 13\Gamma\hat{t}/2)^{2/13}$; the system thus has a “natural” damping time scale of $\hat{t}_{\rm c} = 2/13\Gamma$. Combining the above equations, we can solve directly for $g(e)$,

$$g = \left(\frac{e}{e_0}\right)^{\Gamma} \left(\frac{1-e_0^2}{1-e^2}\right) . \tag{24}$$

We can insert this form back into the eccentricity evolution equation and solve for the time as a function of e,

$$\hat{t} = (1-e_0^2)^{13/2}\, e_0^{-13\Gamma/2} \int_e^{e_0} \frac{de}{e} \, \frac{e^{13\Gamma/2}}{(1-e^2)^{13/2}} \,. \tag{25}$$

In the limit of small eccentricity, we can evaluate the integral to find the approximate solution

$$\hat{t} \approx (1-e_0^2)^{13/2} \, \frac{2}{13\Gamma} \left[1 - (e/e_0)^{13\Gamma/2}\right] . \tag{26}$$

Alternatively, we can find the approximate solution for $e(t)$,

$$e(t) \approx e_0 \Big[1 - \frac{13}{2}\Gamma(1-e_0^2)^{-13/2}\hat{t}\Big]^{2/13\Gamma} \,. \tag{27}$$

This function, in conjunction with the solution for $g(e)$ found above, thus specifies the semi-major axis of the planet as a function of time.

The total evolution time is given by the integral of equation (25) in the limit that $e \to 0$. This time scale can be written in terms of the series

$$\hat{t}_T = (1-e_0^2)^{13/2} \sum_{n=0}^{\infty} \frac{b_n}{n!} \frac{e_0^{2n}}{2n + 13\Gamma/2} \,, \tag{28}$$

where the coefficients b_n are defined through the recursion relation

$$b_{n+1} = (13\Gamma/2 + n)\, b_n \qquad \text{with} \qquad b_0 = 1\,. \tag{29}$$

These analytic results tell us a lot about the long term behavior of the system. Equation (24) implies that the planet does not reach the origin ($g \to 0$) until its eccentricity vanishes. Further, the integral in equation (25) is finite for all $\Gamma > 0$, so the evolution takes place over a finite time (equivalently, the series in equation [28] converges). Finally, equation (28) provides a good working approximation to the total evolutionary time scale, i.e.,

$$\hat{t}_T \approx J^{13} \left[\frac{2}{13\Gamma} + \frac{13\Gamma e_0^2}{4+13\Gamma} + \frac{13\Gamma(2+13\Gamma)e_0^4}{4(8+13\Gamma)} \right] . \tag{30}$$

For many applications, the first term provides an adequate approximation. Notice that the total evolution time is shorter than the characteristic time scale $\tau_{P*} = \tau_0/\Gamma$ (i.e., $\hat{t} = 1/\Gamma$), as defined by equation (5). When the leading order term is a valid approximation, the evolution time is shorter than the characteristic time $\hat{t}_c = 2/13\Gamma$ by an additional factor of $J^{13} = (1-e_0^2)^{13/2}$. For example, if $e_0 = 0.28$ (the median of the observed sample of extrasolar planets), the eccentricity causes the evolutionary time to be shorter by a factor of ~ 2 compared to evolution with no initial eccentricity.

The observed sample of extrasolar planets shows a population of planets with periods of 3 – 4 days, but a deficit of planets with shorter periods. One could, in principle, explain this signature if the shorter period planets were all accreted by their central stars. If this explanation were true, then the total evolution time defined above must be comparable to the stellar ages τ_* (which are typically several Gyr). The requirement that all planets with (initial) semi-major axis less than a_0 are accreted within the stellar age τ_* can be evaluated by using the leading order term for the evolution time (in eq. [30]) and the definitions of τ_0 and Γ (eqs. [2] and [5]). The result takes the form

$$\tau_* \ge 8.6\,\text{Gyr}\ J^{13} \left(\frac{m_P}{m_J}\right)^{-1} \left(\frac{Q_*}{10^6}\right) \left(\frac{a_0}{0.05\text{AU}}\right)^{13/2} , \tag{31}$$

where we have assumed solar properties for the star ($M_* = 1 M_\odot$ and $R_* = 1 R_\odot$). Thus, planets of roughly Jovian mass can be accreted in a typical stellar age (5 Gyr) provided that $Q_* \sim 10^6$ (which is a reasonable value, implied by considerations of eccentricity damping in close binaries, e.g., Hut 1981). Because of the sensitive dependence on semi-major axis a, planets that are accreted spend relatively little time with shorter periods, and, the predicted cutoff in observed period is relatively sharp. Note that $\tau_* \propto a_0^{13/2} \propto P_0^{13/3}$, so that a planet with a 2 day period is accreted 20 times faster than a planet with a 4 day period. For a given age of the stellar population, one would expect to see far more planets in 4 day orbits than in 2 day orbits (if no observational biases are present).

When stellar damping is included, some of the orbital energy is dissipated in the planet, and some is dissipated in the star. For purposes of finding the impact of energy dissipation on the planet (for example, the effect on the planetary radius), we need to find the fraction of the energy that is dissipated in the planet. For this case, we can consider both the energy dissipation and the evolution of semi-major axis to be functions of the eccentricity. Equation (24) specifies the function $g(e)$. The energy dissipated in the planet, as a function of eccentricity e, can be written in the form

$$\Delta E_P = \frac{GM_* m_P}{a_0(1-e_0^2)(2-\Gamma)} \Big[e_0^2 - e^2 (e_0/e)^\Gamma \Big] . \tag{32}$$

This function of eccentricity can be converted into a function of time using the time evolution equations (25 – 31) discussed above. The self-gravitational energy of a Jovian planet can be written $E_{GP} = \eta G m_P^2/R_P$, where the dimensionless parameter $\eta \approx 5/3$ (although the exact value depends on the internal structure and of the planet). We can thus determine the conditions for which a planet will experience a dissipational energy ΔE_P that is comparable to its self-gravity E_{GP}:

$$\frac{\Delta E_P}{E_{GP}} = \frac{M_*}{m_P} \frac{R_P}{a_0} \frac{\Big[e_0^2 - e^2 (e_0/e)^\Gamma \Big]}{\eta(1-e_0^2)(2-\Gamma)} \approx 2.80 \Big(\frac{e_0^2}{1-e_0^2} \Big) \Big(\frac{a_0}{0.05\,\mathrm{AU}} \Big)^{-1} . \tag{33}$$

In the second approximate equality, we have assumed $M_* = 1.0 M_\odot$ and Jovian properties for the planet ($m_P = 1.0 m_J$ and $R_P = 1.0 R_J$). If a hot Jupiter starts with $a_0 = 0.05$ AU and eccentricity $e_0 \approx 0.51$, then the amount of energy dissipated within the planet through tidal interactions is comparable to its self-gravity, so that substantial structural changes can be forced upon the planet. In most cases, however, we expect a smaller eccentricity, so that only a fraction of the planet's self-gravitational energy would be dissipated (as given by eq. [33]). For the median of observed eccentricity of the current sample, $e_M \approx 0.28$, the ratio $\Delta E_P/E_{GP} \approx 0.24$, which is still large enough to be significant.

3.2. Two planet system with stellar damping term

For a two planet system with secular interactions and stellar damping effects, the equation of motion takes the form

$$\frac{df}{d\hat{t}} = - \frac{A f^{5/2}}{(\Pi_0 + f^3 - B f^{7/2})^2 + C f^{17/2}} - \Gamma f^{-11/2} . \tag{34}$$

If we make the same approximations as before (§2.2), where $B, C \ll 1$, the equation of motion can be written in terms of a formal implicit solution of the form

$$\hat{t} = \int_f^1 df\, f^{11/2} \frac{(\Pi_0 + f^3)^2}{\Gamma(\Pi_0 + f^3)^2 + A f^8} . \tag{35}$$

Although the integral cannot be evaluated in terms of elementary functions, a good working approximation can be found if we assume that the first, secular term in the equation of motion is

important only for $f \approx 1$. To find this approximation, we first rewrite the integral in the form

$$\Gamma\hat{t} = \int_f^1 df \left[f^{11/2} - \frac{(A/\Gamma) f^{27/2}}{(\Pi_0 + f^3)^2 + (A/\Gamma) f^8} \right] . \tag{36}$$

We then assume that the second term (which arises from secular perturbations) only is important at the beginning of the evolution with $f \sim 1$, so we can set $f = 1$ in the denominator. The resulting expression for the evolution time becomes

$$\Gamma\hat{t} \approx \frac{2}{13}(1 - f^{13/2}) - \frac{A/\Gamma}{(\Pi_0 + 1)^2 + A/\Gamma} \frac{2}{29}(1 - f^{29/2}) . \tag{37}$$

This approximation is valid as long as the ratio A/Γ is not too large. For example, the error is less than 15% as long as $A/\Gamma < 4$ for $\Pi_0 = 1$. In the extreme limit $\Gamma \ll 1$, where this approximation fails, the additional damping term does not play a role and we can use the no damping solution as a good working approximation. Comparison of this result with that of the previous subsection indicates that in order for the secular interaction of the second planet to play a role, the second term must compete with the first. Almost equivalently, the first term in the evolution equation (34) must compete with the second, i.e., $A \sim \Gamma(1 + \Pi_0)^2$. Inserting typical masses and radii for the planets and star, the requirement for secular interactions to dominate the long term evolution becomes

$$15\, e_2 \left(\frac{a_1}{a_2}\right) \left(\frac{Q_*}{Q_P}\right)^{1/2} \gtrsim 1 + (7.9 \times 10^{-4}) \left(\frac{a_1}{0.05\mathrm{AU}}\right) \left(\frac{a_1}{a_2}\right)^{-3} . \tag{38}$$

Since $a_1 \ll a_2$ for typical systems, this requirement is not met unless $Q_* \gg Q_P$. As a result, the stellar damping effects tend to dominate secular interactions as the inner planet migrates closer to the central star.

For solar systems with sufficiently small semi-major axes and high eccentricities, secular interactions coupled with stellar damping drive the inner planet into the star. To define a region of parameter space for which this effect is important, we take the inner planet to be a hot Jupiter with $m_1 = m_J$, $a = 0.05$ AU, and an eccentricity that cycles through $e_1 = 0$. The orbital elements for the second planet that lead to planetary accretion within 5 Gyr is shown in Figure 2, where we have taken $Q_* = 10^6$. Since the effect depends on the mass of the second planet, we show curves for $m_2 = 1$, 3, and 5 m_J. This plot is thus the analog of that shown in Figure 1, which does not include the stellar damping term. As expected, inclusion of stellar damping effects leads to a larger region of parameter space for which planets can be accreted. The dashed curve delineates the portion of parameter space for which a system with $m_2 = m_J$ would drive its inner planet into the star within 5 Gyr in the absence of relativity.

4. Conclusion

This paper presents solutions for the the long term evolution of four types of planetary systems — one and two planet systems with tidal circularization, both with and without the inclusion of

stellar damping effects. For the two planet systems, planet-planet interactions are modelled using secular interaction theory (MD99) including leading order relativistic corrections (Paper I). The solutions are presented in implicit form; for example, $\hat{t}(f)$ gives the time as a function of the inner planet's semi-major axis, as specified by $f = a_1(t)/a_1(0)$. In the absence of stellar damping, we find exact analytic solutions to the long term evolution, as given by equations (12) and (19). For systems in which the stellar damping term is important, we find approximate analytic solutions given by equations (26) and (37).

In multiple planet systems, secular interactions enhance the accretion of inner planets by the central star, provided that the outer planet has a sufficiently large mass, small semi-major axis, and/or large eccentricity. We have presented a quantitative assessment of the orbital elements of a second planet required to drive a hot Jupiter into the central star within 5 Gyr, for two-planet systems both with (Fig. 2) and without (Fig. 1) stellar damping. General relativity acts to delay the accretion of planets, i.e., the region of parameter space for which the inner planet would be accreted within a given time would be much larger in the absence of relativistic effects (see the dashed curves in Figs. 1 and 2).

For multiple planet systems with hot Jupiters as inner planets, secular interactions tend to give the inner planet nonzero eccentricity. This continual addition of eccentricity, in conjunction with the circularization processes experienced by such planets, leads to large amounts of energy dissipation within the hot Jupiters (eqs. [32] and [33]). Such extreme dissipation, in turn, may lead to mass loss and thereby explain the relatively small masses observed for close planetary companions. This process should be studied in greater detail in the future.

The framework of analysis presented here can be readily applied to individual systems whenever the orbital elements of the constituent planets are known to reasonable accuracy. We maintain an up-to-date catalog of the known extrasolar planets[2] where the time scales derived here are tabulated using the best available fits to the radial velocity data sets. This data base, in conjunction with the analytic solutions presented here, should provide a useful resource for further research on close planetary systems.

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[2]http://www.ucolick.org/~ laugh/

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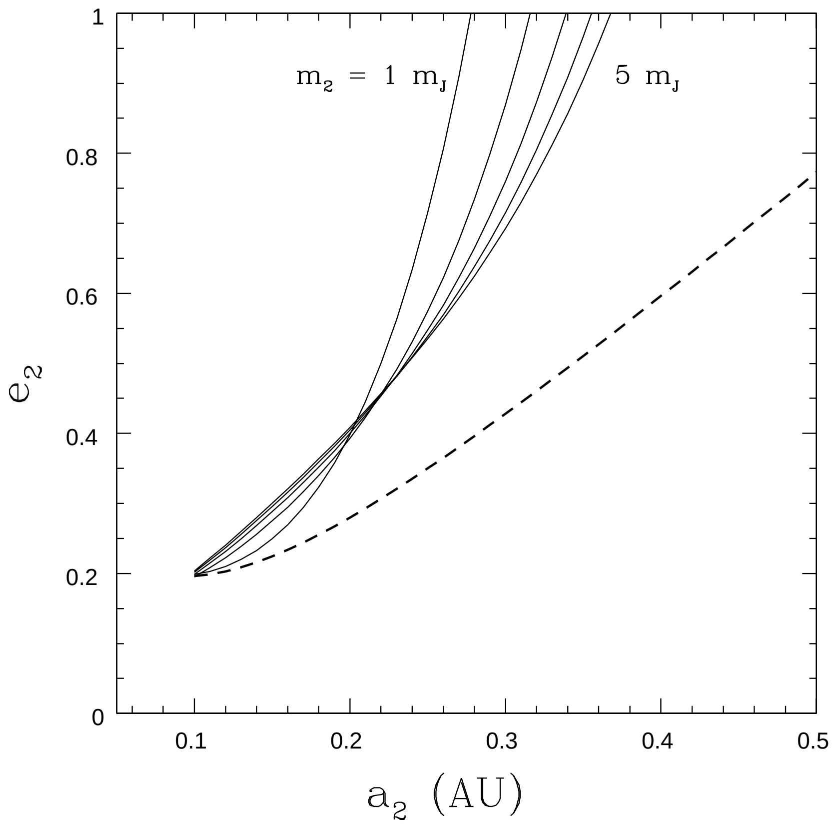

Fig. 1.— For systems that contain a hot Jupiter, this plot shows the orbital elements of the second planet (a_2, e_2) required to drive the hot Jupiter into the stellar photosphere over a time scale of 5 Gyr. All cases assume that the inner planet has mass $m_1 = m_J$, semi-major axis $a_1 = 0.05$ AU, and an eccentricity that cycles through $e_1 = 0$ (where $\langle e \rangle \neq 0$ due to secular interactions with the second planet). The five solid curves correspond to five choices of the mass of the second planet, as labeled. The region of parameter space above the curves corresponds to systems in which the inner planet is driven into the star on shorter time scales $t < 5$ Gyr. The dashed curve delineates the (much larger) region of parameter space for which the inner planet would be driven into the star in the absence of general relativity (for $m_2 = m_J$).

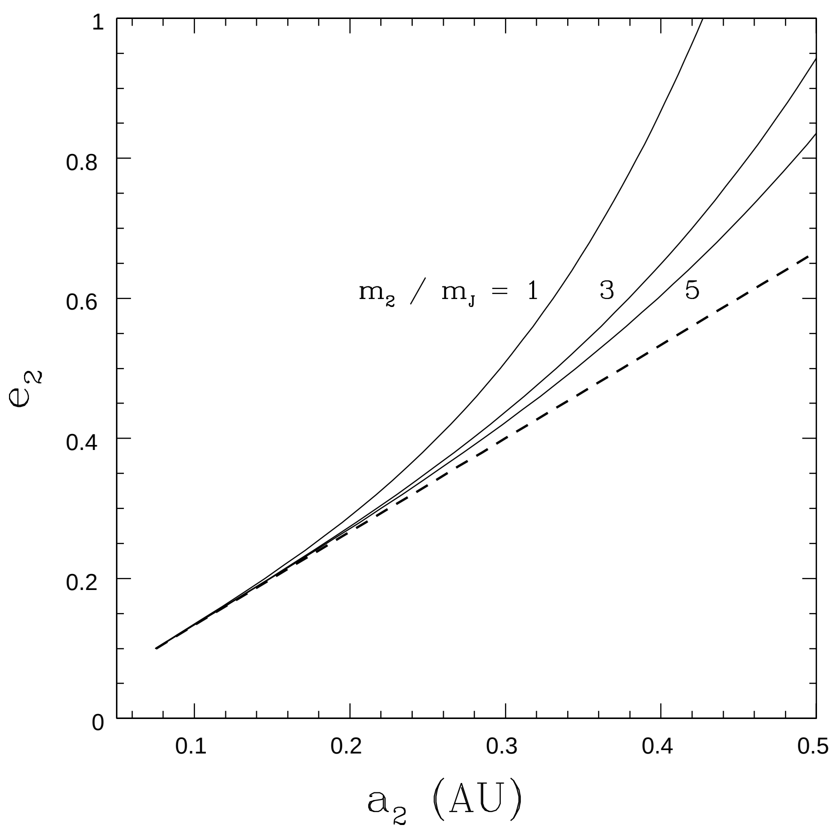

Fig. 2.— For systems that contain a hot Jupiter, this plot shows the orbital elements of the second planet (a_2, e_2) required to drive the inner planet into the star within 5 Gyr, where a stellar damping term is included with $Q_* = 10^6$ (compare with Fig. 1). The inner planet has mass $m_1 = m_J$, semi-major axis $a_1 = 0.05$ AU, and an eccentricity that cycles through $e_1 = 0$ (where $\langle e \rangle \neq 0$ due to secular interactions with the second planet). The three solid curves correspond to different masses of the second planet, as labeled. The region of parameter space above the curves corresponds to systems in which the inner planet is driven into the star on time scales $t < 5$ Gyr. The dashed curve delineates the region of parameter space for which the inner planet would be driven into the star in the absence of general relativity (for $m_2 = m_J$).